OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4753/1

Methods for Advanced Mathematics (C3)

Thursday **8 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

Section A (36 marks)

1 Solve the equation $|3x - 2| = x$. [3]

2 Show that $\displaystyle\int_0^{\frac{1}{6}\pi} x \sin 2x \, dx = \frac{3\sqrt{3} - \pi}{24}$. [6]

3 Fig. 3 shows the curve defined by the equation $y = \arcsin(x - 1)$, for $0 \leqslant x \leqslant 2$.

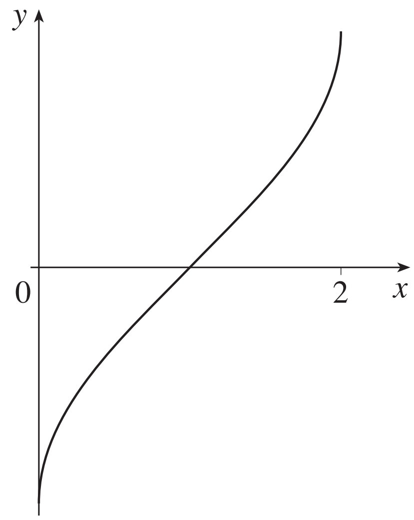

Fig. 3

(i) Find x in terms of y, and show that $\dfrac{dx}{dy} = \cos y$. [3]

(ii) Hence find the exact gradient of the curve at the point where $x = 1.5$. [4]

4 Fig. 4 is a diagram of a garden pond.

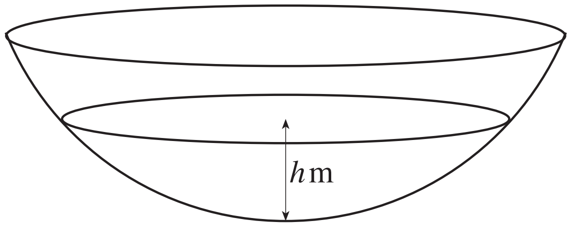

Fig. 4

The volume $V\,\text{m}^3$ of water in the pond when the depth is h metres is given by

$$V = \tfrac{1}{3}\pi h^2(3 - h).$$

(i) Find $\dfrac{\mathrm{d}V}{\mathrm{d}h}$. [2]

Water is poured into the pond at the rate of $0.02\,\text{m}^3$ per minute.

(ii) Find the value of $\dfrac{\mathrm{d}h}{\mathrm{d}t}$ when $h = 0.4$. [4]

5 Positive integers a, b and c are said to form a Pythagorean triple if $a^2 + b^2 = c^2$.

(i) Given that t is an integer greater than 1, show that $2t$, $t^2 - 1$ and $t^2 + 1$ form a Pythagorean triple. [3]

(ii) The two smallest integers of a Pythagorean triple are 20 and 21. Find the third integer.

Use this triple to show that not all Pythagorean triples can be expressed in the form $2t$, $t^2 - 1$ and $t^2 + 1$. [3]

6 The mass M kg of a radioactive material is modelled by the equation

$$M = M_0 \mathrm{e}^{-kt},$$

where M_0 is the initial mass, t is the time in years, and k is a constant which measures the rate of radioactive decay.

(i) Sketch the graph of M against t. [2]

(ii) For Carbon 14, $k = 0.000\,121$. Verify that after 5730 years the mass M has reduced to approximately half the initial mass. [2]

The half-life of a radioactive material is the time taken for its mass to reduce to exactly half the initial mass.

(iii) Show that, in general, the half-life T is given by $T = \dfrac{\ln 2}{k}$. [3]

(iv) Hence find the half-life of Plutonium 239, given that for this material $k = 2.88 \times 10^{-5}$. [1]

[Turn over

Section B (36 marks)

7 Fig. 7 shows the curve $y = \dfrac{x^2 + 3}{x - 1}$. It has a minimum at the point P. The line l is an asymptote to the curve.

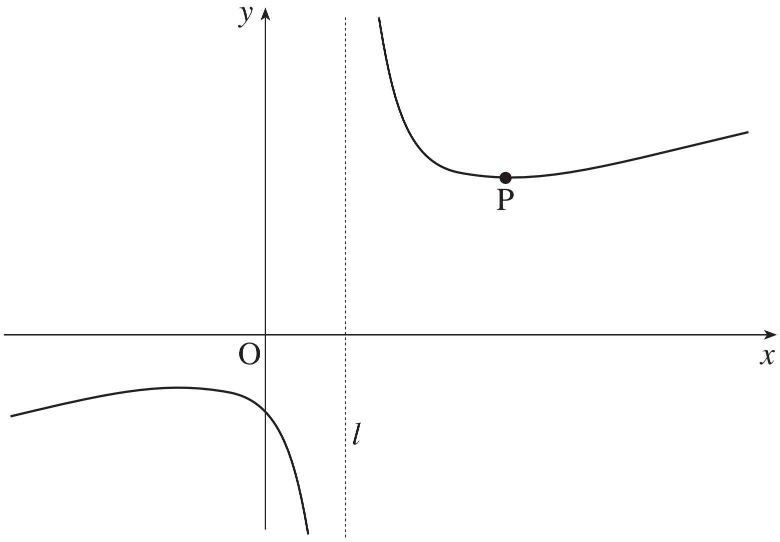

Fig. 7

(i) Write down the equation of the asymptote l. [1]

(ii) Find the coordinates of P. [6]

(iii) Using the substitution $u = x - 1$, show that the area of the region enclosed by the x-axis, the curve and the lines $x = 2$ and $x = 3$ is given by

$$\int_1^2 \left(u + 2 + \frac{4}{u}\right) \mathrm{d}u.$$

Evaluate this area exactly. [7]

(iv) Another curve is defined by the equation $\mathrm{e}^y = \dfrac{x^2 + 3}{x - 1}$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y by differentiating implicitly. Hence find the gradient of this curve at the point where $x = 2$. [4]

8 Fig. 8 shows part of the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \mathrm{e}^{-\frac{1}{5}x} \sin x$, for all x.

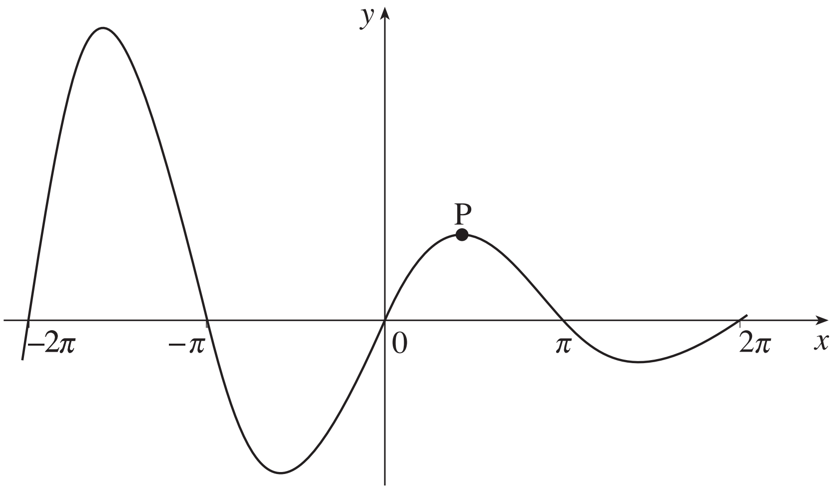

Fig. 8

(i) Sketch the graphs of

(A) $y = \mathrm{f}(2x)$,

(B) $y = \mathrm{f}(x + \pi)$. [4]

(ii) Show that the x-coordinate of the turning point P satisfies the equation $\tan x = 5$.

Hence find the coordinates of P. [6]

(iii) Show that $\mathrm{f}(x+\pi) = -\mathrm{e}^{-\frac{1}{5}\pi}\mathrm{f}(x)$. Hence, using the substitution $u = x - \pi$, show that

$$\int_{\pi}^{2\pi} \mathrm{f}(x)\,\mathrm{d}x = -\mathrm{e}^{-\frac{1}{5}\pi} \int_{0}^{\pi} \mathrm{f}(u)\,\mathrm{d}u.$$

Interpret this result graphically. [You should *not* attempt to integrate $\mathrm{f}(x)$.] [8]